

World-stories and maximality

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1 The world-stories approach

According to many *actualist conceptions of modality*, talk about possible worlds should be reduced to talk about certain sets of propositions called *world-stories*¹.

World-stories are usually characterized as certain particular sets of propositions that are:

- **consistent:** a set of propositions Γ is consistent if, and only if, for any pair of propositions p_1 and p_2 belonging to Γ , then p_1 and p_2 *could be* both true together²
- **maximal:** a set Γ of propositions is maximal if, and only if, for every proposition p , either $p \in \Gamma$ or $\neg p \in \Gamma$.

The basic idea of the world-story theory is that maximality of a set of propositions should mimic the fact that a possible world is, as Kripke says, a *total history*, while the consistency of a set of propositions should mimic the fact that a possible world is ... well ..., just *possible*, namely the fact that whatever is true within a possible world is compatible with whatever else is true within it.

As you surely have noticed the consistency (of a set of propositions) is here defined in modal terms; this is because, as it is well-known, alternative,

¹See, for example, Adams (1974) e Adams (1981).

²Note that the word “together” is essential in the formulation: to just say that p_1 and p_2 could be true *simpliciter* would amount to say the following: “it is possible that p_1 and p_2 are true”. This, however, would be equivalent to say “it is possible that p_1 is true and it is possible that p_2 is true”, which does not capture the intended notion; the role of “together”, in this context, is just that of “blocking” the default distributivity of the modal operator over conjunction. The intended reading is that p_1 and p_2 has to be true within the same possibility.

non modal and “proof-theoretic” conceptions of consistency would not give us the desired results (unless we complicate the theory with a bunch of meaning postulates).

Notice further that, to avoid circularity, the modality involved in such a definition has to be taken as primitive.

The world-story theorist, then, cannot present his theory as a reductive theory of modality; his notion of consistency forced him to say that there is at least a modal notion that is not reducible to a non-modal one and in general that the modal dimension cannot be reduced to the non modal one. The world-story theorist thus needs to conceive himself as a *a modalist*, someone who believes that at least some modal notions are conceptually prior than non-modal ones.

A Russellian conception of propositions is often associated with this view. According to such a conception, propositions are *structured* entities and there are some propositions, called *singular propositions*, that are about a certain object by having that object as a *direct* constituent of the proposition itself. The view that propositions are structured entities (and that their internal structure captures the syntax of a logical language) is sometimes called *structuralism* (and as far as structure is concerned even a Fregean holds a structuralist conception of proposition), the view that propositions may have objects as their constituents is sometimes called *objectualism*. The Russellian conception of propositions is then the result of combining objectualism and structuralism about propositions.

Usually, the world-story theorist is also an *actualist*, a person who believes that there are not *possibilia*, objects that exist (even in some wide, logical sense of ‘exist’) but are not actual.

The Russellian conception of propositions *plus* actualism implies that for the world-story theorist the objectual components of the propositions composing the various world-stories will be just the actual objects. From this it follows that, if the world-story theorist, as an actualist, believes, as it is quite common and auspicious, that not every actual object exist necessarily, then he also believe that the singular propositions composing his world-stories will be contingent existents and, therefore, that also his world-stories will be contingent existents³. This is a quite relevant difference with genuine possible worlds, because genuine possible worlds are taken to be necessary existents.

³There are probably ways to block the inference from the contingency of the actual objects to the contingency of the propositions containing them, and probably also ways to block the inference from the contingency of the singular propositions to the contingency of a set containing them (as an instance of the argument blocking the inference from the contingency of a member of a set to the contingency of the set itself) but we could leave these worries aside.

The association between world-story theory, Structuralism, Objectualism (*i.e.* Russellian conception of propositions) and Actualism is not a forced one. For example, a world-story theorist could in principle not endorse a Russellian conception of propositions (he could be an actualist and a structuralist but not an objectualist) and, probably, does not need also to endorse Actualism (he could be a possibilist, a structuralist and an objectualist).

However, this specific association is surely the most philosophically sensible one and, luckily of us, it is the standard one.

To better characterize a world-story, and in particular its maximality, a way would be specifying them starting first from singular propositions and then using recursive definitions.

There are, however, two methods to obtain this. The differences between the two are not so great but, I think, worth to be pointed out.

First Method: build a maximal and consistent sets of singular propositions and extend recursively the relation of *being a member of* this set.

Second Method: build maximal and consistent sets of singular propositions, call *them world-stories* and define recursively the notion of *truth in a world story*.

The first method would amount to the following (I will use the notation $\ulcorner \Phi^n a_1, \dots a_n \urcorner$ to represent the proposition expressed by the formula $\Phi^n a_1, \dots a_n$ and I will assume that atomic formulas of first-order logic express singular propositions; I will use $\Gamma, \Delta, \Phi, \dots$ as schematic letters for (closed and open) first-order formulas and $\ulcorner \Gamma \urcorner$ for the proposition expressed by Γ)

A set of propositions w is maximal if and only if

- for any actual object $a_1, \dots a_n$ and any property p^n , either $\ulcorner p^n(a_1, \dots a_n) \urcorner \in w$ or $\ulcorner \neg p^n(a_1, \dots a_n) \urcorner \in w$
- $\ulcorner \neg \Gamma \urcorner \in w$ if and only if $\ulcorner \Gamma \urcorner \notin w$
- $\ulcorner (\Gamma \vee \Delta) \urcorner \in w$ if and only if $\ulcorner \Gamma \urcorner \in w$ or $\ulcorner \Delta \urcorner \in w$
- $\ulcorner \forall x \Phi^n(x_1, \dots x_n) \urcorner \in w$ if and only if for any actual objects $a_1, \dots a_n$, $\ulcorner \Phi^n, (a_1, \dots, a_n) \urcorner \in w$
- for any other proposition $\ulcorner \Psi \urcorner$, $\ulcorner \Psi \urcorner \notin w$

Usually, only non-modal propositions are part of a world-story, but if you want to include also modal propositions in such sets, the relevant clause will be the following:

- $\ulcorner \Box \Gamma \urcorner \in w$ if and only if for any world-story w , $\ulcorner \Gamma \urcorner \in w$

The second method would amount to the following: we first define the notion of a *world-story* just as a consistent and maximal set of *singular propositions*. We then define recursively the notion of *truth in a world-story* as follows:

- if $\ulcorner \Gamma \urcorner$ is singular, $\ulcorner \Gamma \urcorner$ is true in a world-story if and only if $\ulcorner \Gamma \urcorner$ belongs to w
- $\ulcorner \neg \Gamma \urcorner$ is true in w if and only if $\ulcorner \Gamma \urcorner$ is not true in w
- $\ulcorner \Gamma \vee \Delta \urcorner$ is true in w if and only if $\ulcorner \Gamma \urcorner$ is true in w or $\ulcorner \Delta \urcorner$ is true in w
- $\ulcorner \forall x \Phi^n(x_1, \dots, x_n) \urcorner$ is true in w if and only if for any a_1, \dots, a_n , $\ulcorner \Phi^n(a_1, \dots, a_n) \urcorner$ is true in w .
- nothing else is true in w .

The relation between a world-story theory built according to the first method and a world-story theory built according to the second is a somewhat typical case of trade-off between ontology and ideology; the second has a much more “austere” version of world-stories (sets containing only singular propositions) but it needs to take as primitive the notion of “truth in a world story”; the first has a much more inflated conception of world-stories but it is able to eliminate the notion of “truth in a world-story” in favour of the notion of “being a member of a world-story”⁴.

Notice that the second method and not the first was the one chosen by R. Carnap to characterize his form of linguistic ersatzism, where a *state-description* was a maximal set of *atomic* sentences and it is the one I favour.⁵

2 The problem of the possible non-existence

Like any other actualist conception of modality, the main problem for the world-story theorist is that of representing, in an actualist acceptable way, *the possible existence of non-actual objects*.

Take a sentence like:

⁴I think, however, while the reduction of one notion to the other is, so to speak, “formally” executed it is just an illusion: the notion of “truth in w ” (be it a world story or a full-bloated possible world) will be always more clear than the notion of belonging of p to w .

⁵Carnap did not need to mention consistency in his system; this, as it is known, was the cause of many problems; probably he believes that all atomic predicates were consistent among them.

(1) There could be a non-actual object.

This is a general sentence, but given the conditions given above, for the proposition expressed by this sentence to be true in a world-story, there must exist a world-story containing a singular proposition having a merely possible object, a non actual one, as a member. An actualist, of course, does not have such an object at hand and therefore is not able to explain why we usually consider this sentence as intuitively true.

For the actualist, the singularity needed to make such a sentence true is, to use K. Fine's words, simply "spurious". As he writes:

for the actualist [...] there can be no instance in virtue of which the sentence is true. The sentence states an *irreducible general possibility*, and no matter how well the individual is described, he can have no specific identity⁶

It is not my aim here to review the ways in which such kind of sentences could be analysed as expressing "irreducible general possibilities". Basically the entire history of actualism as a general philosophical approach to modality could be characterized as the strive to substantiate such a claim.

My aim here is to signal a problem that the world-story theorist already has in representing the *possible non-existence of actual objects*, namely in representing, in an actualist acceptable way, a situation in which, for example, I (an actual object) do not exist.

But what does it mean to represent the possible non existence of something (actually existing)?

My possible non existence, and in general the possible non-existence of something, could be represented either by means of a representation that "encodes" the explicit information that I do not exist, or by means of a representation that does not contain any information about me at all.

For example, assuming that the set of actual objects is composed by me, John and Sam, the following is a representation of a possible situation of the first type, a picture that represents my non-existence *explicitly*:

and this is a representation of a possible situation that represents my non-existence *implicitly*:

In the first case, the information that I do not exist is represented by an intrinsic property of the representation and it is something that I could gather *from within* the picture; in the second case, the information that I do not exist is somewhat inferred by comparison with the actual situation and it is something that I could gather *from the outside*.

⁶See (Fine, 1977, p. 117, my emphasis).

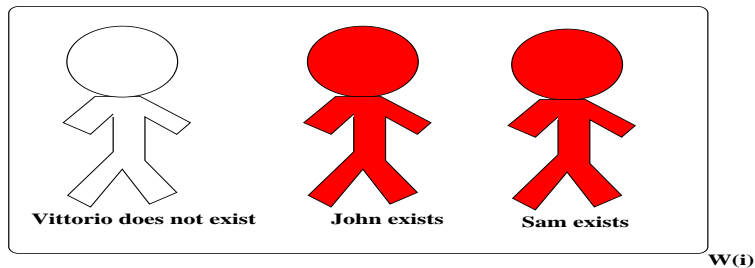


Figure 1: Explicit Possible Non-Existence

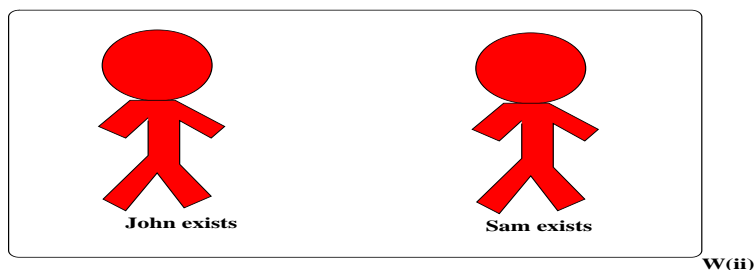


Figure 2: Implicit Possible Non-Existence

The world-story theorist is usually equipped to gathering information from such different sources and in fact he usually enrich his theory with a distinction between what is true *in* a world-story and what is true *at* a world-story.

The intuitive motivation behind the distinction between *truth-in* and *truth-at* a world-story is well explained by K. Fine when he presents the analogous distinction between two notions of propositional truth, the *inner* and the *outer*. Here is what he says:

One should distinguish between two notions of truth for propositions, the *inner*. According to the *outer* notion, a proposition is true in a possible world [in our case in a world-story] regardless of whether it exists in that world; according to the inner notion, a proposition is true in a possible world only if it exists in that world. We may put the distinction in terms of perspective. According to the outer notion, we can stand outside a world and compare the proposition with what goes on in the world in order to ascertain whether it is true. But according to the inner notion, we must first enter the proposition into the world before ascertain

its truth (Fine, 1987, p. 163)

Consider this set of propositions, which is the “propositional representation” of the first figure above:

$$w_i = \{\ulcorner Sam\ exists \urcorner, \ulcorner John\ exists \urcorner, \ulcorner Not: Vittorio\ exists \urcorner\}$$

in this case, the proposition that I do not exist is *true in* the world-story w , the proposition that I do not exist is a member of the set. Consider instead this set of propositions, which is the “propositional representation” of the second figure above:

$$w_{ii} = \{\ulcorner John\ exists \urcorner, \ulcorner Sam\ exists \urcorner\}$$

in such a case, the world-story theorist would say, the proposition that I do not exist is *not* true in w_{ii} but is true *at* w_{ii} .

The distinction between truth-in and truth-at is very useful to the world-story theorist specially because the way in which my possible non-existence is represented in the first figure (or in its propositional counterpart w_i), the way in which my non-existence is represented explicitly, is not a way that an actualist should accept; it is *not actualistically acceptable*.

Prima facie this might sound quite surprising. The only condition that actualism seems to put on the set-theoretic constructions that are conceived to go proxy for genuine possible worlds is that the components of such entities are actual objects.

In particular, the only condition that actualism seems to put on the world-story theorist is that the sets of propositions going proxy for possible worlds should not contain singular propositions directly about merely possible objects.

From this point of view, w_i seems perfectly acceptable. It only contains propositions whose objectual components are actual objects and so it should be taken as a perfectly actualistically acceptable construction.

Why then the way in which my possible non-existence is represented explicitly by the information that I do not exist is problematic for the actualist? Let us see why.

A world-story is either *the* true world-story w^* (a world-story containing all and only true propositions) or it is *a* false world-story (a set of propositions containing at least a false proposition, the negation of a proposition true in the true world-story).

A false world-story, however, has the modal property of *being possibly the true set of proposition*. Had things gone differently, instead of the world-story w^* (the true world-story), another world-story, say w_i , would have been true.

Had w_i been true, a singular proposition about me would have been true⁷. But had w_i been true, I would have not existed, because it would have been true the proposition that I do not exist. But then, had w_i been true, a singular proposition about a non-actual object would have been true, and this is something that an actualist should deny.

An actualist should not simply affirm the weak thesis that nothing singular in nature could be true of merely possible objects; this way of expressing actualism in fact leave open the possibility that something singular in nature could be true of actual objects even in those possible situations in which they do not exist. From the stand-point of that situation, the actual objects in questions would be just merely possible objects.

The actualist should therefore affirm the stronger thesis that *necessarily*, nothing singular in nature could be true of merely possible objects. From this, it should follow that, had things gone differently, had some actual object a not exist, nothing singular in nature could have been true of a too. In a situation in which I do not exist, there should be nothing true (or false) about me, neither the proposition that I do not exist.

Note that this thesis is also implied by the general assumptions by which a world-story theory is build; as I said before, a world-story theorist is a structuralist, an objectualist and an actualist; if he also is an *essentialist about his structuralism*, that is if he believes that structured propositions have their constituents essentially, then structuralism, objectualism and actualism implies a thesis usually call *existentialism*:

Existentialism: If a proposition Γ exists and it is about a , then had a not existed, neither Γ would have existed

The explicit way of representing the possible non existence of an actual objects is not, then, compatible with the world-story theory. This is because, the standard definition of maximality is only able to generate world-stories in which my possible non existence is represented explicitly.

If this is the situation, however, there seems to be a big problem with the standard notion of maximality given before.

According to that definition, a world-story is maximal if and only if for *any actual object* a_1, \dots, a_n and property P^n , either $P^n a_1, \dots, a_n$ or $\neg P^n a_1, \dots, a_n$. The result is that every world story will have something directly about *every* actual object; if a is an actual object, the standard conception of maximality will generate a world-story containing a proposition saying *of a* that it does not exist.

⁷Sometimes it is argued that the result of negating a singular proposition is not a singular proposition, ...

The standard conception of maximality will give us world-stories where everything is specified, even those informations that could be obtained “by omission” and derived by means of the notion of truth-at.

Another conception of maximality is needed.

A first step in order to solve the problem is to emphasise a distinction between two ways in which the notion of “description of an alternative courses of actuality” could be intended:

- a description of actuality with respect to an alternative course of it
- a description of an alternative course of actuality

Leaving aside the problem of describing alternative courses of actuality where more objects than the actual ones exist (a problem that the world-story theorist has in any case), the problem is thus that the standard conception of maximality is only able to give us the first kind of description (*i.e.*, a description that, for any actual object, tells us what happens to it with respect to an alternative course of events), whereas what we need to have (what an actualist should looking for) is a conception of maximality able to give us a description of actuality of the second kind (*i.e.*, a description of an alternative course of actuality that specifies, in a maximal and consistent way, only what happens to the actual objects existing in that alternative course of events).

I propose to call the first kind of maximality **global maximality** and the second kind **local maximality**. The problems in representing my possible non existence derives from the fact that the world-story theorist usually has a global conception of maximality.

I will, in the next section, try to characterise the local conception of maximality.

3 Local maximality

In order to characterize local maximality, I propose first to characterize the notion of *actual object that would have existed had a certain (set of) proposition(s) been true*. The idea is basically that we need a systematic way, given a certain set of false propositions, to determine what objects would exist, had that set of propositions been true. We could then define maximality only with respect to those objects.

Let us consider the simplest case, namely that of a set of propositions w_i whose only component is $\lceil Pa \rceil$. Assume that $\lceil Pa \rceil$ is actually false and ask yourself: what actual objects would have existed had w been true? In this

case the answer is quite easy and unequivocal: a is the actual object that would have existed, had w been true.

Things are not so simple, however. Sometimes (probably the majority of times) the truth of a proposition Γ is compatible with the existence of distinct sets of actual objects; hence, the answer to the question “What actual objects could have existed, had Γ been true” might turn out quite equivocal.

Consider, for example, the proposition expressed by $\exists x(x \neq a)$ and assume that such a proposition is a member of a consistent set of propositions w_{ii} whose other member is $\lceil Pa \rceil$. Assume that the set of actual objects is just $\{a, b, c\}$.

Now, what actual objects would have existed had w_{ii} been true? The truth of the proposition expressed by Pa requires the existence of a but the truth of the proposition expressed by $\exists x(x \neq a)$ is compatible with the existence of distinct sets of actual objects, namely $@_{w_1} = \{a, b\}$, $@_{w_2} = \{a, c\}$ and $@_{w_3} = \{a, b, c\}$. Had w_{ii} been true, the actual objects that would have existed had w_{ii} been true would have been either $@_{w_1}$ or $@_{w_2}$ or $@_{w_3}$. Call the distinct sets of actual objects compatible with the truth of a certain set of propositions, the $@_w$ -sets.

Now, my idea is that we can answer unequivocally to the question of what actual objects are compatible with the truth of a certain set of proposition w only relatively to an $@_w$ -set.

The question *What actual objects would have existed had w been true?* should be reinterpreted as the question *what actual objects, relatively, to an $@_w$ -set would have existed, had w been true?* With the expression *the actual objects that, relatively to $@_{w_i}$, would have existed, had w realized* I will simply denote all the (actual) objects belonging to $@_{w_i}$.

With the notion of $@_w$ -set at our disposal, we are now ready to define a notion of local maximality – that I will call $@_w$ -maximality – for a set of actually existing proposition w .

I will use Γ_i or Δ_i to refer to the propositions Γ and Δ such that all their objectual components (if there are any) are elements of the set i . I will use an expression p^n to refer to any primitive, qualitative property⁸ and I will use an expression like Φ_i to refer either to a primitive property or to any non primitive property Φ such that its only objectual components (if there are any) are elements of the set i .

A set of propositions w is $@_{w_i}$ -maximal, for some i , if and only if

⁸By qualitative property I simply means a property whose linguistic formulation does not mention individuals.

- for any n -ary property p^n and for any actual object $a_1, \dots, a_n \in @_{w_i}$, either $\lceil p^n(a_1, \dots, a_n) \rceil \in w$ or $\lceil \neg p^n(a_1, \dots, a_n) \rceil \in w$.
- $\lceil \neg \Gamma_{@_{w_i}} \rceil \in w$ if and only if $\Gamma_{@_{w_i}} \notin w$
- $\lceil \Gamma_{@_{w_i}} \vee \Delta_{@_{w_i}} \rceil \in w$ if and only if $\lceil \Gamma_{@_{w_i}} \rceil \in w$ or $\lceil \Delta_{@_{w_i}} \rceil \in w$
- $\lceil \forall x \Phi_{@_{w_i}} x \rceil \in w$ if and only if for any $a \in @_{w_i}$, $\lceil \Phi_{@_{w_i}} a \rceil \in w$
- for any other proposition $\lceil \Psi \rceil$, $\lceil \Psi \rceil \notin w$.

With the notion of $@_{w_i}$ -maximality at hand, we can now define a world-stories as follows:

If w is a set of proposition, w is a *world-story* if and only if w is consistent and, for some i , $@_{w_i}$ -maximal.

The new conception of maximality allows us to generate world-stories where my possible non-existence is represented simply by the lack of any proposition having me as a constituent and is therefore coherent with the general principles inspiring the world-stories approach to modality.

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